REPORT 1138

STUDY OF INADVERTENT SPEED INCREASES IN TRANSPORT OPERATION [1]

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SUMMARY

Some factors relating to inadvertent speed and Mach number increases in transport operation are discussed with the object of indicating the manner in which they might vary with different qualities of the airplane and the minimum margins required to guard against reaching unsafe values. The speed increments and the margins required under several assumed conditions are investigated. The results indicate that, on a percentage basis, smaller margins should be required of high-speed airplanes than of low-speed airplanes to prevent overspeeding in inadvertent maneuvers. The possibility of exceeding placard speed in prolonged descents is illustrated by computations for typical transport airplanes. Equations are suggested that allow estimates to be made of the necessary speed margins.

INTRODUCTION

In order to guard against inadvertent increases in airspeed, flight regulations limit the effective airplane cruising speed to a fixed percentage (80 percent) of the design or demonstrated speed. For lack of better information, the same limit has been applied to the Mach number. As a result of these regulations, low-altitude propeller-driven airplanes tend to be limited by the indicated-airspeed placard so that a margin of 20 percent is maintained on the speed for structurally safe flight but with the possibility of a greater margin on the Mach number where adverse compressibility effects occur. On the other hand high-altitude high-speed jet-powered transports might be expected to be limited in cruising by the Mach number placard which results in a 20 percent margin on Mach number and a greater margin on the structurally safe indicated speed.

Plans for the development of turbojet transports have renewed interest in the problem of selecting satisfactory limits for airplane operating speeds that will insure against exceeding values of either Mach number or the dynamic pressure for which the airplane can be expected to remain controllable and structurally sound. In order for such transports to be economically feasible, however, they must necessarily be operated nearer the maximum level-flight speed than the older propeller-driven airplanes and excessive margins cannot be tolerated.

For these reasons reexamination of the problem of selecting the limiting operating speeds appears desirable. This report presents an analysis for determining the margins required under several assumed conditions and is confined mainly to the physical aspects of the problem.

SYMBOLS

The following symbols are used throughout the present report:

A	aspect ratio, b^2/S
a	speed of sound, fps
b	wing span, ft
C_D	airplane drag coefficient, Drag/qS
C_{D_0}	profile-drag coefficient
C_L	airplane lift coefficient, Lift/qS
C_{L_α}	airplane lift-curve slope per radian
$C_{L_{\alpha_t}}$	tailplane lift-curve slope per radian
c	wing chord, ft
C_{m_0}	pitching-moment coefficient of wing-fuselage combination at zero lift
α_t	tail angle of attack, radians
d	horizontal distance from given object, or distance between tail-off aerodynamic center and center of gravity, ft
$\frac{d\epsilon}{d\alpha}$	downwash factor
e	span efficiency factor
f, f_0	compressibility factors defined in reference 1
g	acceleration of gravity, 32.2 ft/sec^2
M	Mach number, V/a
q	dynamic pressure, $\frac{1}{2}\rho V^2$, lb/sq ft
S	wing area, sq ft
S_t	tail area, sq ft
T	engine thrust, lb
t	time, sec
U	true gust velocity, fps
V	true velocity, fps except with subscript mph
V_c	calibrated airspeed (the airspeed related to differential pressure by the accepted standard adiabatic formula used in the calibration of differential-pressure airspeed indicators and equal to true airspeed for standard sea-level conditions), fps
W	airplane weight, lb
w_p	weight of shifted payload, lb
x_p	distance through which payload is shifted, ft
x_t	distance from airplane center of gravity to tail hinge line, ft
L/D	airplane lift-drag ratio
h	altitude, ft

[1] Supersedes NACA TN 2638, "Study of Inadvertent Speed Increases in Transport Operation" by Henry A. Pearson, 1952.

Δh	altitude change, ft
Δn	incremental load factor
ΔV	velocity increase, fps
$\Delta h/\Delta t$ or dh/dt	rate of descent, fps
$\Delta V/\Delta t$ or dV/dt	rate of change of velocity, ft/sec^2
α	angle of attack, radians
δ	elevator angle, radians
γ	flight-path angle, radians
ρ	mass density of air, slugs/cu ft
ρ_0	mass density of air at sea level, slugs/cu ft

CONDITIONS FOR WHICH SPEED MARGINS ARE REQUIRED

Some of the conditions which may be considered as leading to inadvertent increases in airspeed are listed as follows:

(1) Increases in speed and Mach number resulting from maneuvers made either to avoid obstacles or from a sudden failure of automatic pilot or booster system

(2) Increases in speed and Mach number resulting from encountering gusts during cruising

(3) Increase in speed and Mach number due to a forward shift in passengers or payload

(4) Mach number margin required to permit maneuvering without reaching the buffeting boundary

(5) Mach number changes resulting from traversing areas with temperature inversions

(6) Increase in speed and Mach number associated with carrying out a planned descent from altitude.

AVOIDANCE OF OBSTACLES

If an airplane were required to execute a rapid push-down pull-up maneuver in order to pass under an obstacle on a collision course, an increment in speed and Mach number would be gained during both the push-down part of the maneuver and the recovery to level flight. For example, if a small altitude loss were necessary in order safely to clear an obstacle, an equal altitude change would be required to return to level flight providing both the push-down and pull-up were made with equal rapidity. If all of the potential energy represented by the combined altitude change were converted into kinetic energy, the equation for this limiting case would be

$$\frac{1}{2}\frac{W}{g}\left[(V+\Delta V)^2-V^2\right]=W\Delta h \tag{1}$$

Expanding $(V+\Delta V)^2$, dropping the second-order term, and dividing through by V^2 reduces the expression to

$$\frac{\Delta V}{V}=\frac{g\,\Delta h}{V^2} \tag{2}$$

From the relation $V=Ma$ another form of equation (2) is

$$\frac{\Delta M}{M}=\frac{g\,\Delta h}{M^2a^2} \tag{2a}$$

Thus, to a first approximation, the maximum possible percentage increase in velocity incurred in clearing a given obstacle would vary inversely as the square of the initial speed or Mach number. Since the obstacle to be avoided would most likely be another airplane, the minimum altitude loss would be of the order of 50 feet and the minimum total altitude change, including an equally rapid recovery, would be of the order of 100 feet. From equation (2) the percentage change in speed for this amount of altitude change is determined to be a 1-percent increase for an airplane traveling 388 miles per hour and a 5-percent increase for an airplane traveling 173 miles per hour. Thus, speed increments resulting from avoiding collisions would be of consequence only if the total altitude loss were larger, as would be the case if the pilot tried to clear by more than 50 feet or failed to recover as rapidly after clearing the obstacle. The margin in speed required to guard against this possibility should vary inversely with the cruising speed.

Of some interest, perhaps, in such a maneuver is the small distance over which the flight path can be changed without imposing large loads on the airplane. If it is assumed that a load-factor increment of Δn could be instantaneously applied, the greatest deviation which can be made to the flight path would be given by the following equation

$$\Delta h=\frac{gd^2}{2V^2}\Delta n=\frac{gt^2}{2}\Delta n \tag{3}$$

where

Δn acceleration increment in g units

t time during which load increment is applied

d distance at which object to be cleared is first sighted

For example, if an object were initially seen at the distance corresponding to that traveled in 2 seconds and a load-factor increment of 2 were instantaneously applied, a deviation of only 128 feet in the altitude could be made under this assumption. The fact that the pilot could not react immediately plus the fact that the airplane does not respond instantaneously to an instantaneous elevator impulse may reduce the value in this case to about one-quarter, or from 128 to 32 feet.

It appears from this example that the increase in speed due to avoiding obstacles would be important only if the minimum altitude changes were made larger; however, unless a collision path was recognized in time, the altitude changes involved would be small.

AUTOPILOT OR BOOSTER FAILURE

The altitude change and increase in speed if an automatic pilot or a booster system were to fail suddenly are also related by equations (1) to (3). In this case, however, both the time during which the acceleration increment acts and the pilot reaction time may be longer because of the unexpectedness of the failure, with the result that both the altitude losses and the increases in speed would be larger. The percentage increase would again vary inversely with the square of the initial velocity and would be given by the equations

$$\frac{\Delta V}{V}=\frac{g^2t^2\Delta n}{2V^2} \tag{4a}$$

$$\frac{\Delta M}{M}=\frac{g^2t^2\Delta n}{2a^2M^2} \tag{4b}$$

If the sudden failure is assumed to impose a load-factor increment Δn of -2 (a value that would normally be within the strength limits of the wing), equations (4) may be rewritten

$$\frac{\Delta V}{V} \cong 1000 \left(\frac{t}{V}\right)^2 \tag{5a}$$

$$\frac{\Delta V}{M} \cong 1000 \left(\frac{t}{Ma}\right)^2 \tag{5b}$$

so that, if a maximum fractional increase of not more than 0.1 in speed is to be obtained, the ratio t/V should be less than 1/100. For example, for a transport traveling 350 feet per second, recovery should be started within $3\frac{1}{2}$ seconds. Inasmuch as the time for initiating a recovery would be a constant and independent of the initial speed, it appears that a smaller speed margin should be required for high-speed airplanes than for low-speed airplanes to guard against an autopilot failure.

SYMMETRICAL NORMAL GUST

Equation (3) also serves as a starting point for arriving at estimates of the margins required to cover the speed gained in encountering gusts normal or parallel to the flight path. Assume that an airplane with neutral stability encounters a negative gust normal to the flight path for which the intensity increases linearly to a peak in 10 chords ($10c$), or a distance H, and decreases thereafter linearly to 0 in the same distance. This type of gust-intensity distribution is one commonly investigated in gust-load calculations. If unsteady lift and alleviation defects are omitted, the following simple load-factor relations apply

$$\Delta n_{max} = \frac{C_{L_\alpha} \rho U V}{2 W/S} = 2\, \Delta n_{av} \tag{6}$$

The altitude loss for this case is

$$\Delta h = \frac{\Delta n_{av} g t^2}{2}$$

where

$$t = \frac{20c}{V}$$

so that

$$\Delta h = \frac{1}{2} \frac{C_{L_\alpha} \rho U V g}{4 W/S} \frac{(20c)^2}{V^2} \tag{7}$$

and from equations (2) and (7)

$$\frac{\Delta V}{V} = \frac{\Delta M}{M} = \frac{1}{8} C_{L_\alpha} \frac{U}{V^3} \frac{\rho g^2}{W/S} (20c)^2 = \frac{50 C_{L_\alpha} U \rho g^2}{W/S} \frac{c^2}{V^3} \tag{8}$$

In this case the percentage increase in speed or Mach number varies inversely with the cube of the initial airspeed or Mach number, directly with the square of the distance occupied by the gusts, and directly with the gust velocity. If typical maximum values are substituted in equation (6), it may be seen that the percentage increase in speed in encountering a single normal gust is likely to be small, providing the airspeed is not initially so low that the gust causes the airplane to stall. For a succession of down gusts the increase in speed would be larger than for a single down gust but even so the percentage gain would be less with the faster airplane than with the slower one.

SYMMETRICAL HORIZONTAL GUST

For horizontal gusts that occur in level flight, the increases in airspeed and Mach number would be instantaneous and are likely to be relatively higher than those due to the normal gust. In this case the fractional increase in airspeed or Mach number is given directly by the equation

$$\frac{\Delta V}{V} = \frac{\Delta M}{M} = \frac{U}{V} \tag{9}$$

A horizontal gust would usually be of relatively short duration and would not be expected to be too critical in its effects; however, in a descent from altitude through an area of wind shear, more prolonged gust effects might be encountered. Although the magnitudes of the velocity and the gradients in such an area of wind shear are not definitely known, it appears that the gust velocity for use in equation (9) is either less than or, at most, equal to the value of 50 feet per second commonly used.

SHIFTING PAYLOAD

A gain in speed or in Mach number can occur if a shift in weight were to occur as when several passengers walk from the rear to the front of the cabin and the pilot fails to retrim the airplane. If it is assumed (a) that the aerodynamic coefficients are linear and do not vary with Mach number over the range being considered, (b) that the speed gain occurs in an atmosphere of constant density equal to that of the initial altitude, and (c) that the pilot does not move either the elevator or throttle before a new equilibrium condition is established, then the following derivation may be made. By designating the initial conditions by the subscript 1 and new trim conditions by subscript 2, the equilibrium equations can be written as

$$W = C_{L_\alpha} \alpha_1 q_1 S \tag{10}$$

$$\left(C_{m_0} \frac{c}{x_t} + C_{L_\alpha} \alpha_1 \frac{d_1}{x_t} - C_{L_{\alpha_t}} \alpha_{t_1} \frac{S_t}{S} - C_{L_{\delta_t}} \delta_1 \frac{S_t}{S}\right) q_1 S = 0 \tag{11}$$

After a change in moment $w_p x_p$ due to a shift in payload the new equilibrium equations are

$$W = C_{L_\alpha} \alpha_2 q_2 S \tag{12}$$

$$\left(C_{m_0} \frac{c}{x_t} + C_{L_\alpha} \alpha_2 \frac{d_2}{x_t} - C_{L_{\alpha_t}} \alpha_{t_2} \frac{S_t}{S} - C_{L_{\delta_t}} \delta_2 \frac{S_t}{S}\right) q_2 S = 0 \tag{13}$$

where the distance d_2 is related to the distance d_1 through the equation

$$d_2 = d_1 - \frac{w_p x_p}{W} \tag{14}$$

If equation (13) is subtracted from equation (11) and if $qS \neq 0$, there is obtained

$$C_{L_\alpha} \left(\alpha_1 \frac{d_1}{x_t} - \alpha_2 \frac{d_2}{x_t}\right) - C_{L_{\alpha_t}} \frac{S_t}{S} (\alpha_{t_1} - \alpha_{t_2}) = 0 \tag{15}$$

By noting that

$$\alpha_{t_1}=\left(1-\frac{d\epsilon}{d\alpha}\right)\alpha_1$$

and that

$$\frac{\alpha_2}{\alpha_1}=\frac{\alpha_{t_2}}{\alpha_{t_1}}$$

equation (15) can be written as

$$\left[C_{L_\alpha}\frac{d_1}{x_t}-C_{L_{\alpha_t}}\frac{S_t}{S}\left(1-\frac{d\epsilon}{d\alpha}\right)\right]\alpha_1-\left[C_{L_\alpha}\frac{d_2}{x_t}-C_{L_{\alpha_t}}\frac{S_t}{S}\left(1-\frac{d\epsilon}{d\alpha}\right)\right]\alpha_2=0 \quad (16)$$

By using equations (10) and (12), equation (16) can be rearranged as follows:

$$\frac{\alpha_1}{\alpha_2}=\frac{q_1}{q_2}=\frac{V_2^2}{V_1^2}=\frac{M_2^2}{M_1^2}=\frac{\left[C_{L_\alpha}\frac{d_2}{x_t}-C_{L_{\alpha_t}}\frac{S_t}{S}\left(1-\frac{d\epsilon}{d\alpha}\right)\right]}{\left[C_{L_\alpha}\frac{d_1}{x_t}-C_{L_{\alpha_t}}\frac{S_t}{S}\left(1-\frac{d\epsilon}{d\alpha}\right)\right]} \quad (17)$$

Substituting d_2 (from eq. (14)) into equation (17) and rearranging gives

$$\frac{V_2}{V_1}=\frac{M_2}{M_1}=\sqrt{1+\frac{\left(\frac{w_p}{W}\right)\frac{x_p}{x_t}}{\frac{C_{L_{\alpha_t}}}{C_{L_\alpha}}\frac{S_t}{S}\left(1-\frac{d\epsilon}{d\alpha}\right)-\frac{d_1}{x_t}}} \quad (18)$$

If V_2 is set equal to $V_1+\Delta V$ and if second-order terms are neglected,

$$\frac{\Delta V}{V_1}=\frac{\Delta M}{M_1}\cong\frac{\frac{1}{2}\left(\frac{w_p}{W}\right)\left(\frac{x_p}{x_t}\right)}{\frac{C_{L_{\alpha_t}}}{C_{L_\alpha}}\frac{S_t}{S}\left(1-\frac{d\epsilon}{d\alpha}\right)-\frac{d_1}{x_t}} \quad (19)$$

Equation (19) indicates that the percentage increase in speed is independent of the initial speed and depends on the aerodynamic and geometric characteristics of the airplane. It shows that the ratio $\Delta V/V$ increases as the ratio of the change in trim moment to the airplane moment about the hinge line increases $\left(\frac{w_p x_p}{W x_t}\right)$ and that the ratio $\Delta V/V$ increases as the static stability (given by the denominator) decreases. The speed increase given by equation (19) is the equilibrium value reached under the assumption made and would correspond to a condition in which the airplane is in a slight but steady glide angle. The transient values of $\Delta V/V$ may be lower or higher than the trim value given by equation (19) depending upon when the pilot takes appropriate action to retrim. The largest value of $\Delta V/V$ should occur at a time about equal to one-quarter of the period of the phugoid oscillation of the airplane $\left(\text{roughly } \frac{V_{mph}}{16} \text{ sec}\right)$. Since this motion is lightly damped, the maximum value reached could be very nearly equal to twice that given by equation (19).

As an example of the quantities involved for the case of a typical present-day four-engine transport, a change in either $\Delta V/V$ or $\Delta M/M$ equal to 0.025 would be obtained from equation (19) if 500 pounds of passengers moved from the most rearward position to the front seats.

MARGINS REQUIRED FOR MANEUVERING

Most current airplanes capable of operating in or near the transonic speed range are limited to operation below what is commonly called a buffeting boundary. A typical buffeting boundary for a fighter airplane, designated airplane 1, is shown in figure 1. The data for this curve are taken from reference 2. The part of the curve to the right of the dashed vertical line is associated with a separation caused by compressibility on some main component of the airplane; whereas that part of the curve to the left of the vertical line is the usual $C_{L_{max}}$ curve. Although the boundary to the right of the vertical line can be and has been crossed during special tests with small military airplanes, to do so not only subjects the airplane to large oscillating forces but also places it in a region where stability and control difficulties occur. At present a quantitative evaluation of either the forces or handling qualities of airplanes beyond or at this boundary is not possible. For these reasons, transport airplanes should not be operated at the boundary and some margin appears to be necessary to permit mild maneuvers at cruising speed without the possibility of reaching the buffeting boundary.

In the high-speed cruising range, which is mainly of interest in the present study, the slopes of the available buffeting boundaries are relatively steep. Thus, some idea of the margins which should be maintained can be obtained, even though, as stated previously, the boundary cannot be predicted too accurately. Information on the buffeting boundaries was obtained by analyzing the results for several airplanes, including that given in figure 1, to show the margin required in Mach number in order that a steady 45° bank could be executed at cruising speed without exceeding the boundary. Execution of a 45° bank without loss of altitude requires that the airplane lift coefficient be increased by the factor $\sqrt{2}$. Thus, if, for example, in figure 1 the original

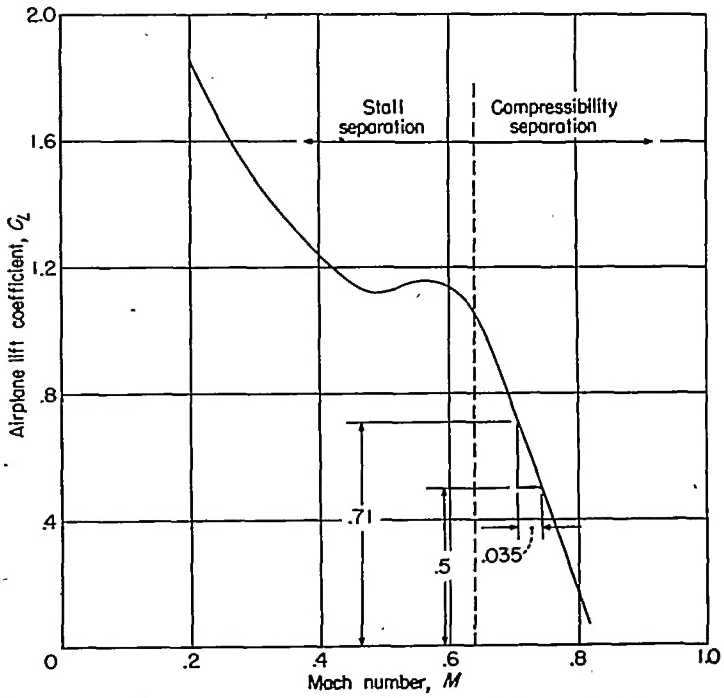


FIGURE 1.—Typical buffeting boundary for high-speed fighter airplane 1.

cruising lift coefficient C_L were 0.5 the lift coefficient in the bank would be approximately 0.71. As shown in figure 1, the margin in Mach number required would then be about 0.035. This point is illustrated in figure 2 by the circular symbol on the curve for airplane 1. Repeating this procedure at other lift coefficients and for other airplanes gave the results shown in figure 2 in which the abscissa represents the over-all airplane C_L at cruising speed and the ordinate is the Mach number margin from the buffeting boundary that should be maintained if mild maneuvering at cruising speed is to be permitted. In general, the Mach number at which the airplane would buffet in level flight is best determined from flight demonstration tests and the margins of figure 2 would be applied to this Mach number. For the present at least, a conservative estimate of the margin required to permit maneuvering appears to be given by any one of the following:

$$\Delta M=\frac{C_L}{12} \text{ allows } 30° \text{ bank}$$

$$\Delta M=\frac{C_L}{10} \text{ allows } 45° \text{ bank}$$

$$\Delta M=\frac{C_L}{7} \text{ allows } 60° \text{ bank}$$

The relation $\Delta M=\frac{C_L}{10}$ represents an envelope of most of the data shown in figure 2 for the 45° bank. The relations given for other angles of bank were determined in a similar manner.

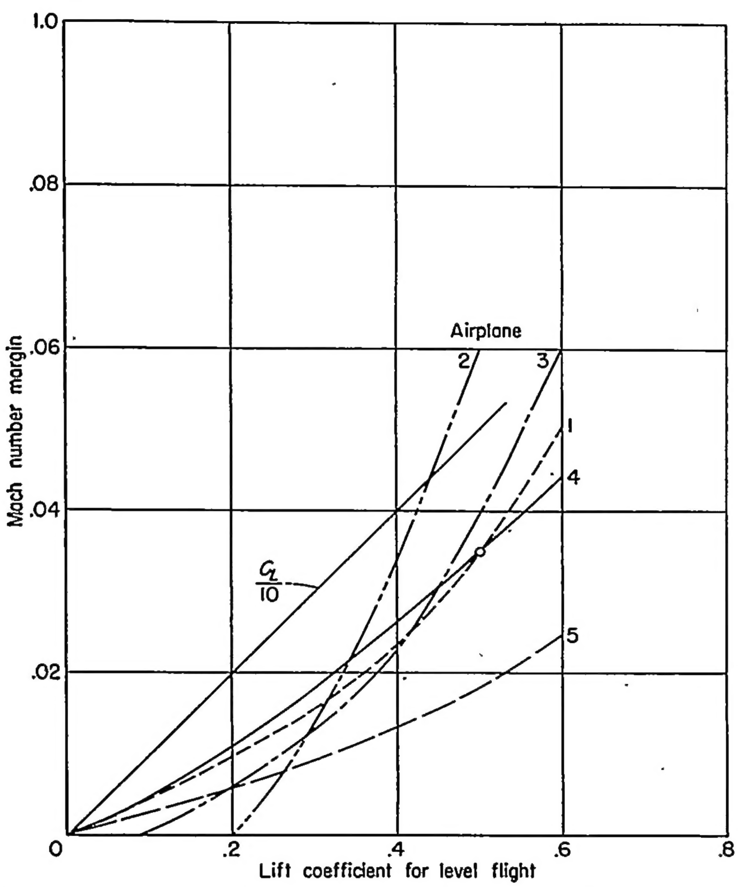


FIGURE 2.—Mach number margin required to execute 45° bank without crossing the buffet boundary.

MARGIN REQUIRED FOR TEMPERATURE INVERSION

Temperature inversions are known to exist in the atmosphere and the altitude range over which inversions may occur varies from hundreds to thousands of feet. Within such inversions fairly localized gradients of 10° F per thousand feet are not uncommon. A change in the temperature of 10° F corresponds to a change of about 1.3 percent in the speed of sound.

During steep descents with small airplanes operating near critical Mach number, adverse compressibility effects such as buffeting or stability changes are sometimes inadvertently encountered. These occurrences have been correlated with measured temperature inversions so that, in some types of research testing, a margin of about 0.015 in Mach number has been necessary in order to avoid inadvertently reaching the buffeting boundary. Since the airplanes on which such experience has been obtained had critical Mach numbers of about 0.75, a suitable margin to guard against the effects of a temperature inversion during a descent may be obtained from the relation

$$\frac{\Delta M}{M}=0.02$$

The constant 0.02 in this relation would be associated with a somewhat larger temperature gradient than the 10° F mentioned earlier.

SPEED GAINS DURING PROLONGED DESCENTS

Statistical data have shown that the probability of exceeding the placard speeds is greatest in prolonged descents. Some of the reasons for such overspeeding (exceeding placard speed), such as meeting schedules or encountering an emergency, are obvious, whereas other less obvious reasons could conceivably be linked with the operation of either the engine or cabin pressurization system.

Overspeeding in the case of an emergency cannot be rationalized as a pilot would take whatever risks were required. Even introducing automatically operated devices such as brake flaps would not positively prevent overspeeding unless these flaps provided sufficient braking to keep the terminal velocity in a steep dive below the placard value.

Because jet engines must be operated at a higher percentage of power at high altitudes than piston engines in order to avoid a "flame out," this characteristic could offer an excuse for overspeeding in a descent in case the engines could not be restarted easily. In such a case a pilot, if pressed for time, might not tolerate the low rates of descent which would be forced on him by operating the engine at a relatively high percentage of power. Similarly, an airplane having a cabin pressurized by an exhaust-driven turbosupercharger might also offer an excuse for overspeeding since some engine power would be required during a descent in order to maintain cabin pressure. The obvious remedies in both these instances would be to provide positive means of restarting jet engines in flight and the avoidance of pressurization by exhaust-driven superchargers; otherwise, air brakes would be necessary in order to compensate for the undesirable engine thrust and the weight component in a descent.

In addition to these somewhat unusual and possibly outmoded cases, overspeeding might also occur if the pilot were

to follow some fixed plan of descent without making due allowances for airplane characteristics. Conceivably, a descent from altitude could be made according to a number of predetermined plans such as: at constant indicated airspeed, at constant Mach number, at a constant true airspeed, at constant glide-path angle, at constant rate of change of absolute altitude, or at constant rate of change of cabin pressure altitude. In each of these plans there would be a steady decrease in the potential and total energy involved. All plans, however, would be characterized by conditions which could be treated by equations (1) to (3). For instance, the initial phase of the descent before reaching steady conditions may be considered transient as may be any subsequent deviations from the main path due to overcontrolling or inattentiveness on the part of the pilot. These additional speed changes over and above that called for by the adopted plan can be treated as before by using equation (2) or its equivalent

$$\frac{dV}{dt}=\frac{-g}{V}\frac{dh}{dt} \tag{20}$$

In equation (2) the maximum possible percentage increase for a given Δh below the intended descent path varied inversely as the square of the average speed along the path. Equation (20) indicates that the rate of change of speed is directly proportional to the rate of descent and inversely proportional to the speed; thus the higher the initial speed the greater the time available for the pilot to prevent a unit increase in speed by detecting and checking a unit rate of descent above the intended value.

The equations of motion for some of the plans of descent which might be used are given in general form, where the asterisk is used with the symbols to identify the parameters held constant.

For a descent at a constant small glide-path angle for which the weight may be considered equal to the lift, the equation of motion is

$$\frac{dV}{dt}=g\left(\sin\gamma^*+\frac{T}{W}-\frac{1}{L/D}\right) \tag{21}$$

Since by definition

$$\sin\gamma=\frac{dh/dt}{V}=\frac{dh/dt}{Ma} \tag{22}$$

and, from reference 1,

$$V_c=V\frac{f_0}{f}\sqrt{\frac{\rho}{\rho_0}} \tag{23}$$

the equation of motion for a descent at a constant calibrated airspeed is (calibrated airspeed is the pilots' indicated airspeed when the airspeed system has no error)

$$\frac{dV}{dt}=g\left(\frac{dh/dt}{V_c^*}\frac{f_0}{f}\sqrt{\frac{\rho}{\rho_0}}+\frac{T}{W}-\frac{1}{L/D}\right) \tag{24}$$

Similarly, for a descent at constant Mach number the equation is

$$\frac{dV}{dt}=g\left(\frac{dh/dt}{M^*a}+\frac{T}{W}-\frac{1}{L/D}\right) \tag{25}$$

and for a constant rate of change of altitude based on standard conditions

$$\frac{dV}{dt}=g\left[\frac{(dh/dt)^*}{V}+\frac{T}{W}-\frac{1}{L/D}\right] \tag{26}$$

Alternate forms of equations (21) to (26) may be obtained by substituting relations involving the coefficients of lift and drag

$$D=C_DqS=C_D\frac{\rho}{2}V^2S=C_D\frac{\rho a^2}{2}M^2S \tag{27}$$

where C_D may be expressed as

$$C_D=C_{D_0}+\frac{C_L^2}{e\pi A}=C_{D_0}+\left(\frac{W\cos\gamma}{qS}\right)^2\frac{1}{e\pi A} \tag{28}$$

In general T/W will be some function of V and h for a given engine speed or throttle setting and L/D will be a function of Mach number M and C_L.

ILLUSTRATIVE EXAMPLES AND APPLICATION OF RESULTS

AIRPLANE CHARACTERISTICS AND CONDITIONS

In order to integrate equations (21), (24), (25), and (26) to obtain the speed gain with a given descent plan, a step-by-step solution must be made. Since several of the principal variables are either nonlinear or are complicated functions of other variables, an infinite number of solutions would exist so that no general charts can be given. However, in order to illustrate the potential speed gains that may occur in following various plans for descent, examples are given for three typical transport airplanes designated airplanes A, B, and C. The airplane characteristics and conditions assumed to illustrate the application of the formulas are summarized in table I.

Airplane A is representative of a propeller-driven airplane of about ten years ago with a nonpressurized cabin. In the example, this airplane is assumed to start a descent from

TABLE I.—PERTINENT AIRPLANE CHARACTERISTICS AND CONDITIONS USED IN EXAMPLES

Quantity	Airplane		
	A	B	C
Weight, lb	25,000	85,000	125,000
Wing loading, lb/sq ft	25	55	75
Thrust/Weight	$18/V_{mph}$	$23.1/V_{mph}$	$19{,}000-0.3H$
Initial conditions:			
Altitude, ft	10,000	20,000	30,000
V, mph	200	350	500
M	0.272	0.495	0.738
V_c, mph	172.6	259.2	230.1
q, lb/sq ft	75.5	166.5	238.0
γ, radians	[a] 0.0284	0.0324	0.0227
	[b] 0.0568	------	0.0454
$\Delta h/\Delta t$, ft/min	[a] 500	1000	1000
	[b] 1000	------	2000

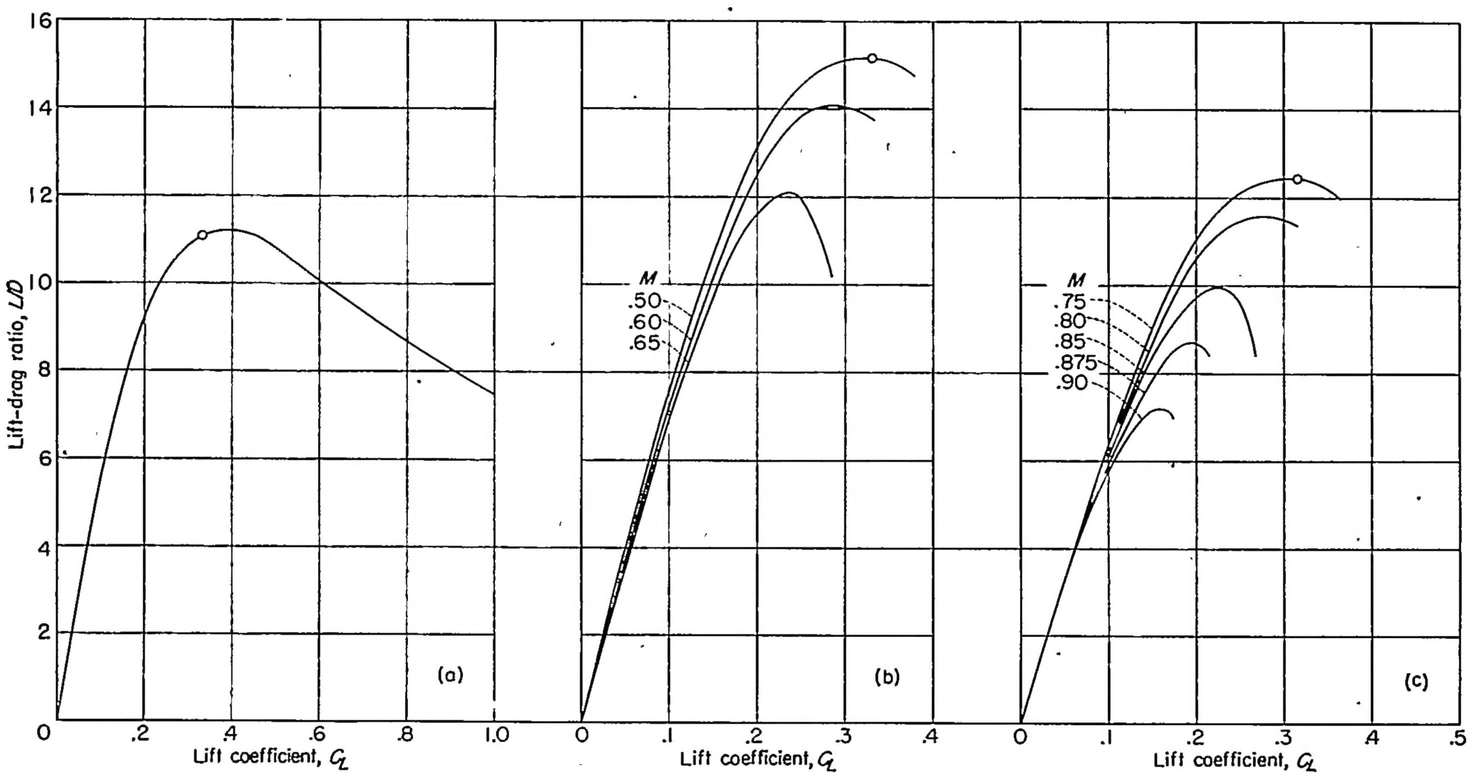
[a] Descent shown both with engine power assumed constant and with zero thrust.
[b] Descent shown with zero thrust only.

10,000 feet with an initial true airspeed of 200 miles per hour for two thrust conditions. In one case the engine power and propeller efficiency are assumed to be constant during the descent so that the thrust-weight ratio varies inversely with the speed according to the relation $\frac{T}{W}=\frac{18}{V_{mph}}$. In the other case the thrust is assumed to be zero. The variation of L/D with airplane lift coefficient C_L is given in figure 3 (a) where the initial condition from which the computations were started is represented by the circular point. Since the speed range of airplane A is quite low, no Mach number effects on the L/D curve were assumed. Also since it was assumed that no cabin pressurization was used, the rate of change of altitude was kept low for the various descent plans.

Airplane B is a pressurized-cabin, propeller-driven airplane typical of some present-day transports. For this airplane the descent was assumed to start at 20,000 feet from a true cruising speed of 350 miles per hour. Rates of descent of 1000 and 2000 feet per minute were assumed. As with airplane A, two cases were considered: one with zero power and one in which the engine power and propeller efficiency were assumed constant in the descent. For this case, the total thrust-weight ratio was given by $\frac{T}{W}=\frac{23.1}{V_{mph}}$. The assumed variation of L/D with C_L and Mach number is given by the curves in figure 3 (b), which were derived from tests of a current transport configuration. From supplementary curves it was established that critical Mach number occurred around $M=0.65$; the initial conditions are represented by the point on the $M=0.50$ curve.

Airplane C used in the examples is an assumed swept-wing, pressurized-cabin, turbojet airplane capable of cruising at 30,000 feet at 500 miles per hour or a Mach number of about 0.75. The L/D curves for this airplane are given in figure 3 (c). The critical Mach number for airplane C was established as being around 0.81. For the case of flight with power on, the variation of thrust for the speed and altitude range of interest was assumed to be given by the equation $T=19000-0.3h$. Results of wind-tunnel model tests and jet engine tests were used to estimate the L/D and thrust relations.

For airplane A, which was not pressurized, the value $\frac{dh}{dt}=500$ feet per minute is slightly above the upper limit permissible in current practice regarding the effect of rate of change of pressure on passenger comfort, whereas the 1000 feet per minute might apply with only crew members aboard. For airplane B in which the cabin was assumed to be pressurized to 10,000 feet, the rate of descent was chosen as 1,000 feet per minute which would give a total descent time of 20 minutes. With this rate, a total time of about 23 minutes would be required to raise the cabin pressure from 10,000 feet to sea level at a cabin rate not exceeding that corresponding to a change of 0.4 inch of mercury per minute. (A rate not exceeding 0.4 inch of mercury per minute represents current practice. This rate corresponds to values of dh/dt equal to 370, 500, 700, and 1000 feet per minute at sea level, 10,000, 20,000, and 30,000 feet, respectively.) For airplane C also with the 10,000-foot pressurized cabin, descent rates of 1,000 and 2,000 feet per miute were chosen.



(a) Airplane A. (b) Airplane B. (c) Airplane C.

FIGURE 3.—Variation of lift-drag ratio with lift coefficient for airplanes of example.

At the lower rate, the cabin and outside pressure would just be equalized shortly before landing, whereas at the higher rate some ground time would be required.

CALCULATIONS FOR SPECIFIC DESCENT PLANS

Calculations were made for the various specific descent plans for airplanes A, B, and C and the results are presented in figures 4 to 6. In the computations the transition from the cruising condition to the specific condition planned for the descent was assumed to be instantaneous. In the figures, the altitude is plotted against calibrated airspeed for each plan of descent. Ticks are added to each curve to indicate the elapsed time in minutes. The values of dh/dt used in the calculations were based on values intended to provide reasonable passenger comfort and adequate cabin pressurization.

Descent at constant V_c.—A descent at constant calibrated airspeed would be represented in figures 4 to 6 by vertical lines. For the range of conditions considered, constant calibrated airspeed corresponds closely to a constant dynamic pressure q. Therefore, during such a descent the airplane lift coefficient and lift-drag ratios would remain nearly constant except for Mach number effects on these quantities.

The difference between a descent at a constant calibrated airspeed and one at a constant dynamic pressure q, if such a descent could be made, may be obtained by noting the deviation of line D in figures 4 to 6 from a vertical line through the initial point. From the deviations shown, it appears that in a descent at constant calibrated airspeed the dynamic pressure would be expected to increase slightly.

Descent at constant M.—Curve C of figures 4 to 6 shows that the descent at constant M would result in an increase in calibrated speed and hence in the dynamic pressure q. On a percentage basis, the increase in calibrated speed is successively greater with airplanes A, B, and C mainly because the altitude range covered is greater. Regardless of the initial cruising speed, the increase in true airspeed during a constant Mach number descent in a standard atmosphere would not exceed 10 percent; however, for airplane C, the value of q would increase about $3\frac{1}{2}$ times during the descent from 30,000 to 2,000 feet. This increase in q for descent at constant Mach number would probably be excessive from structural considerations, so that this plan is a less practical one than the descent at constant calibrated airspeed.

Descent plan	With thrust	Zero thrust
$\gamma = 0.0284$ radian	A	E
$\gamma = 0.0568$ radian		F
$\frac{dh}{dt} = 500$ ft/min	B	G
$\frac{dh}{dt} = 1000$ ft/min		H
$M = 0.272$	C	C
$q = 75.5$ lb/sq ft	D	D

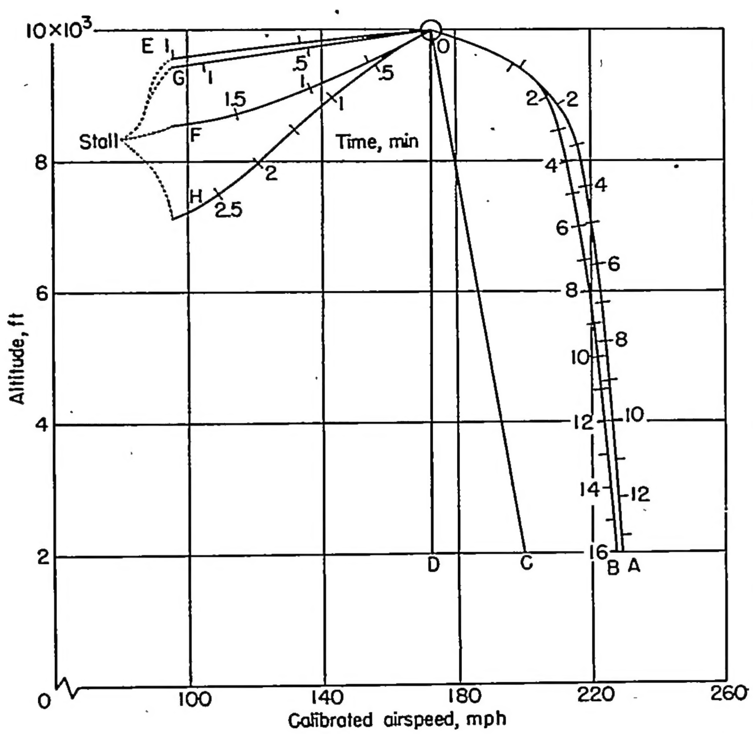


FIGURE 4.—Velocity-altitude relations for various descent plans with airplane A.

Descent plan	
$\gamma = 0.0324$ radian	A
$\frac{dh}{dt} = 1000$ ft/min	B
$M = 0.495$	C
$q = 166.5$ lb/sq ft	D

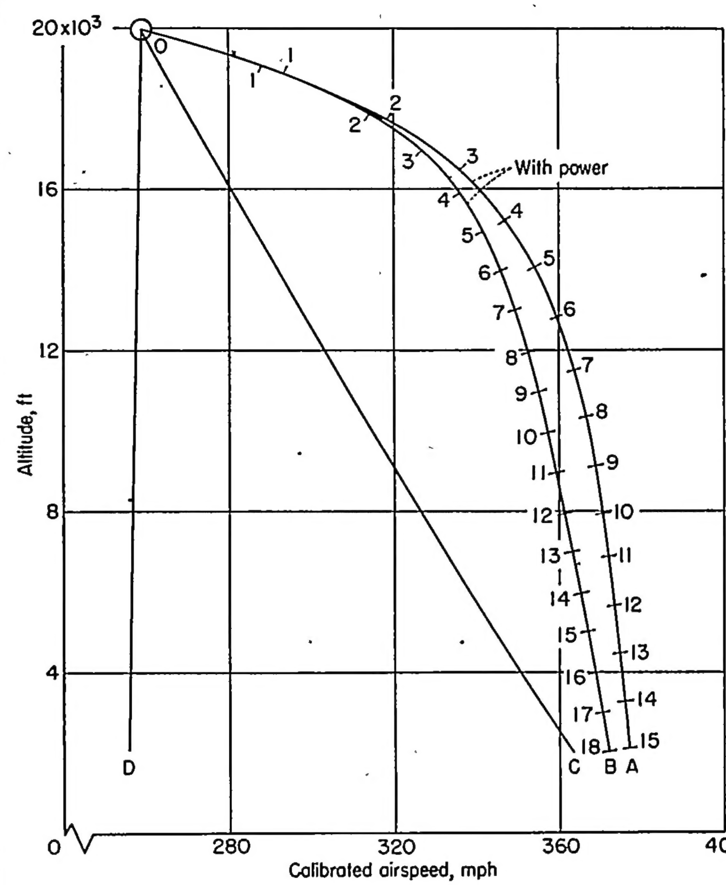


FIGURE 5.—Velocity-altitude relations for various descent plans with airplane B.

The curves given in figures 4 to 6 for the descent at constant M apply to either the zero thrust or power-on condition, since the pilot would adjust the throttle and glide angle as required in order to maintain the values selected. In fact, equations (24) and (25) cannot be integrated for the power-on cases considered in this report unless throttling or some brake devices are used.

Descent at constant rate of change of altitude.—The computations for airplane A (curve B, fig. 4) show that, with constant power setting, the calibrated speed in a descent at constant rate of change of altitude would be increased by about 24 percent in 5 minutes, of which about half would occur within the first minute. For the power-off case (curves G and H) the airplane would stall in trying to maintain a constant rate of change of altitude of either 500 or 1000 feet per minute. Thus it appears that, in the event of sudden engine failure, rates of descent higher than 1000 feet per minute would necessarily prevail regardless of passenger comfort.

The fact that the increases in speed measured in present-day transport operations have generally been less than 10 percent means that some throttling is used during the descent. For airplane A, computations would show that if the engines were immediately throttled to about two-thirds of the cruising power (that is, about four-ninths of rated power) a descent could be made at about 500 feet per minute without a substantial increase in speed.

Descent plan	
γ = 0.0227 radian	A
γ = 0.0454 radian	E
$\frac{dh}{dt}$ = 1000 ft/min	B
$\frac{dh}{dt}$ = 2000 ft/min	F
M = 0.738	C
q = 238 lb/sq ft	D

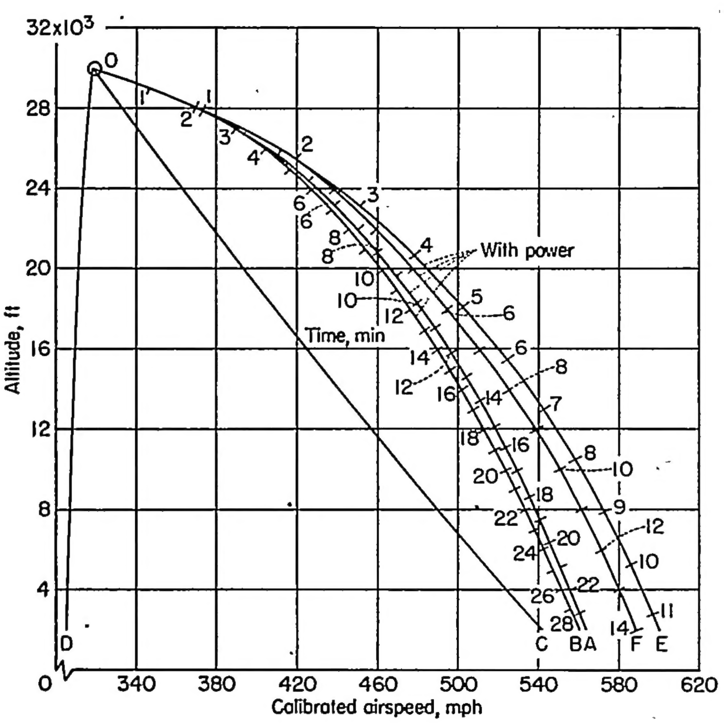


FIGURE 6.—Velocity-altitude relations for various descent plans with airplane C.

The increase in speed for airplanes B and C at the constant rates of descent chosen are, on a percentage basis, about the same as for airplane A with about half the final maximum increase occurring in the first minute. Thus, it appears that, if the present placard speed which limits cruising operation to 80 percent of the design or demonstrated speed is to be raised with future transports, provisions must be made for reducing the "effective" airplane L/D ratio either by engine throttling or by use of aerodynamic braking.

Descent along constant flight-path angle.—The curves labeled A, E, and F in figures 4 to 6 for descents at constant flight-path angle indicate slightly greater increases in speed and consequently greater rates of descent than the curves for constant rate of descent (curves B, G, and H), even though the flight-path angle γ was selected on the basis that it be equal to the rate of descent divided by the initial airspeed. Of interest are the small glide-path angles involved which seldom exceed more than 2°. These small angles are in approximate agreement with statistical measurements which have seldom indicated glide-path angles in a descent of over 5°.

COMPOSITE PLAN

Although the specific plans discussed would probably not be followed throughout a descent without modifications, they are useful in indicating the problem and in pointing out safe procedures to be followed.

It is possible that, unless the cabin were capable of being pressurized to sea-level pressure up to the highest cruising altitude, passenger comfort not only could influence the type of descent plan but also could affect the placard speeds. As stated previously, present practice is to limit the rate of change of cabin pressure to about 0.4 inches of mercury per minute which corresponds to a value of $\frac{dh}{dt}$=1000 feet per minute at 30,000 feet and 370 feet per minute at sea level.

For structural reasons future transports will continue to be designed to withstand some maximum dynamic pressure q or its equivalent in airspeed. This maximum dynamic pressure could either be one which is arbitrarily selected as a design point or one to which the airplane must be demonstrated. Transport airplanes of the immediate future will also be limited by some Mach number which is not to be exceeded if stability and control troubles as well as buffeting are to be avoided. These considerations will in general require that a composite plan of descent be adopted.

In order to illustrate these limits some of the results for airplane C given in figures 3 (c) and 6 can be used. It is assumed that wind-tunnel tests of a model or flight demonstrations have shown that, at low lift coefficients, adverse compressibility effects begin at M=0.81 (represented by C') and that this value of M should not be exceeded. The structure is assumed to have been designed or demonstrated

to withstand the loads at a dynamic pressure corresponding to a calibrated airspeed of 422 miles per hour. These extreme operational limits which should not be exceeded are shown in figure 7 by the heavy dashed line having two segments and labeled "design limit." The short upper segment is a part of the curve representing the 0.81 Mach number limit, whereas the lower segment is a part of the curve representing the dynamic-pressure limit. For comparison, lines B and C from figure 6 are also shown from which it is seen that neither the descent at a constant value of $\frac{dh}{dt}=1000$ nor at a constant $M=0.738$ could be followed throughout without exceeding these limits.

REQUIRED MARGINS

In order to allow for the possibility of inadvertent increases in speed and Mach number, some margin from the limits shown by the heavy dashed line is required. Although equations have already been given from which the margins required under a single condition may be obtained, the question arises as to the probability of separate events occurring together. The speed gains resulting from avoiding obstacles, from encountering normal down gusts, and from shifting payload have been omitted in determining the required margins because either they have been shown to be small or the probability of these events occurring simultaneously with other more important events is remote. Deliberate overspeeding beyond established placard speeds has also been omitted because of its psychological aspect. With these possibilities eliminated, the combinations of events that might have reasonable probability of occurrence are

(1) A mild maneuver during descent at operational speed (30° bank) in a region of temperature inversion where horizontal gusts of moderate size (15 fps) exist

(2) Autopilot failure resulting in $-2g$ increment in load factor with 5 seconds delay in recovery in a region of moderate horizontal gusts.

Consideration of these possibilities indicates that a guide to the required margins in Mach number from the operational limits might be obtained from the equation

$$\Delta M=\frac{C_L}{12}+15\left(\frac{M}{V}\right)_{cruise}+0.02M_{crit}+0.02 \tag{29}$$

where the first term is the margin on the buffet boundary at the airplane cruising lift coefficient, the second term is a margin allowing for a 15-foot-per-second horizontal gust at cruising speed, the third term allows for a possible temperature inversion, and the last term takes into consideration the spread in critical Mach number for a series of airplanes of the same type.

The reduction from the design indicated airspeed is based on the second possibility and would be given by an equation of the type

$$\Delta V=45+\frac{25000}{V_{design}} \tag{30}$$

where the first term is a combined one allowing for horizontal gusts and variations between airplanes and the second term allows for the possibility of an autopilot failure. In equation (30) V_{design} is considered to be the true airspeed in feet per second corresponding to the design indicated airspeed at the lowest altitude at which the autopilot would be used.

If these suggested equations were applied to airplane C, the reduction from the critical Mach number of 0.810 would be 0.072 or 8.9 percent and would yield a maximum operational Mach number of 0.738. If 10,000 feet is assumed to be the lowest altitude in which flight with the autopilot would occur, the reduction from the true design airspeed of 484 miles per hour would be 80 feet per second or about 55 miles per hour. This new airspeed (484 mph – 55 mph) would correspond to an indicated airspeed of 374 miles per hour, which represents a margin of 11.5 percent on the design indicated speed of 422 miles per hour. The operational limits obtained in this manner are given by the solid heavy line in figure 7.

It should be remembered that the constants appearing in equations (29) and (30) for calculating the required margins are estimated and should be checked at the first opportunity with flight experiences.

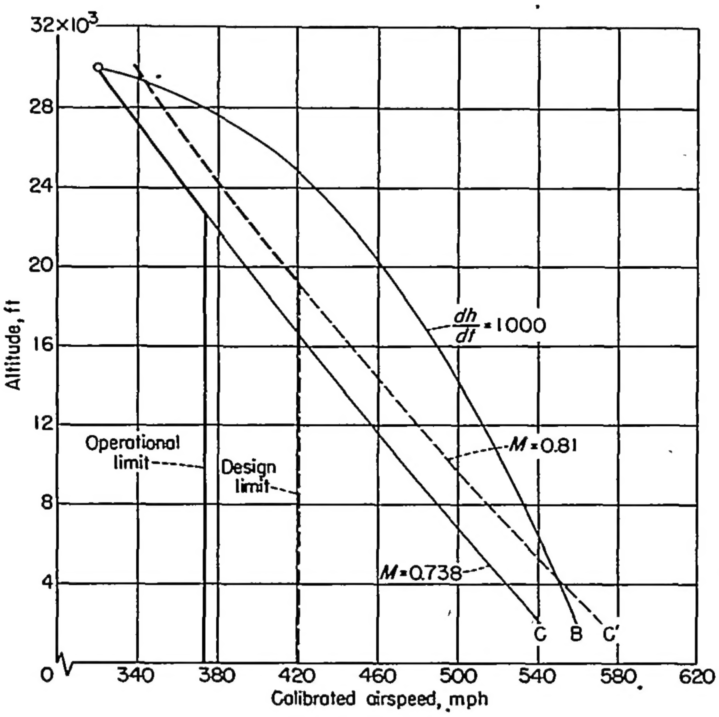


FIGURE 7.—Operational and design limits for composite descent. (Airplane C.)

CONCLUDING REMARKS

As a result of a study to estimate the speed margins that should be allowed to provide for inadvertent speed increases in transport operation, the following trends are indicated:

1. As the cruising speeds of transports increase, the percentage margins required to avoid inadvertent speed gains caused by gusts, autopilot failure, and so forth should decrease.

2. The descent plan of future transport airplanes will probably be a composite one in which the Mach number will be used to furnish the limit in the beginning of the descent and the indicated airspeed will furnish the limit during the later stages.

3. The necessity of including aerodynamic-braking devices will become increasingly important with future transports if reasonable rates of descent are to be attained without large increases in either the airspeed or Mach number. The size and the projected area of such brakes should, however, be coordinated with the requirements of passenger comfort and the type of cabin pressurization used.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., *November 16, 1951.*

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